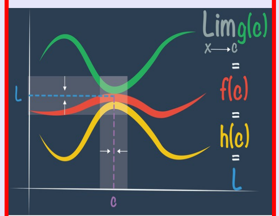
Calculus I

Lecture 4



Feb 19-8:47 AM

Class Quiz 4

Evaluate the following limits

1) $\lim_{x\to 0}\left[\cos x + \tan x - \sin\left(\frac{\pi}{2}-x\right)\right]$

$= \cos 0 + \tan 0 - \sin\left(\frac{\pi}{2}-0\right) = 1 + 0 - 1 = \boxed{0}$

$\pi/2$

2) $\lim_{x\to -4}\frac{x^2+4x}{x^2-16} = \lim_{x\to 4}\frac{x(\cancel{x+4})}{(\cancel{x+4})(x-4)}$

$= \frac{(-4)^2+4(-4)}{(-4)^2-16}$

$= \frac{16-16}{16-16} = \frac{0}{0}$ I.F.

$= \lim_{x\to 4}\frac{x}{x-4}$

$= \frac{-4}{-4-4} = \frac{-4}{-8}$

$= \boxed{\frac{1}{2}}$

3) $\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1} = \frac{\frac{1}{1}-1}{1-1}$

$= \frac{0}{0}$ I.F.

LCD $= x$

$= \lim_{x\to 1}\frac{\overset{-1}{\cancel{1-x}}}{x(\cancel{x-1})}$

$= \lim_{x\to 1}\frac{-1}{x} = \frac{-1}{1} = \boxed{-1}$

Feb 19-7:57 AM

Evaluate

1) $$\lim_{x\to 2} \frac{x^2-5x+6}{x^2-6x+8} = \frac{2^2-5(2)+6}{2^2-6(2)+8} = \frac{0}{0} \text{ I.F.}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x-4)} = \lim_{x\to 2} \frac{x-3}{x-4} = \frac{2-3}{2-4} = \frac{-1}{-2} = \boxed{\frac{1}{2}}$$

2) $$\lim_{x\to 0} \frac{\frac{1}{x+1}-1}{x} = \frac{\frac{1}{0+1}-1}{0} = \frac{1-1}{0} = \frac{0}{0} \text{ I.F.}$$

LCD $= x+1$

$$= \lim_{x\to 0} \frac{\cancel{(x+1)}\cdot\frac{1}{\cancel{x+1}} - (x+1)\cdot 1}{(x+1)x} = \lim_{x\to 0} \frac{\cancel{1}-x-\cancel{1}}{(x+1)x}$$

$$= \lim_{x\to 0} \frac{-\cancel{x}}{(x+1)\cancel{x}} = \lim_{x\to 0} \frac{-1}{x+1} = \frac{-1}{0+1} = \boxed{-1}$$

Feb 24-9:07 AM

Evaluate $(x^2)^2-4^2$

1) $$\lim_{x\to -2} \frac{x^4-16}{x^2-4} = \frac{(-2)^4-16}{(-2)^2-4} = \frac{16-16}{4-4} = \frac{0}{0} \text{ I.F.}$$

$$= \lim_{x\to -2} \frac{(x^2+4)\cancel{(x^2-4)}}{\cancel{x^2-4}} = \lim_{x\to -2}(x^2+4) = (-2)^2+4 = \boxed{8}$$

2) $$\lim_{x\to 3} \frac{x^2-9}{x^3-27} = \frac{3^2-9}{3^3-27} = \frac{0}{0} \text{ I.F.}$$

A^3-B^3

$$= \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x^2+3x+9)} = \lim_{x\to 3} \frac{x+3}{x^2+3x+9} = \frac{3+3}{3^2+3(3)+9}$$

$$= \frac{6}{27} = \boxed{\frac{2}{9}}$$

Feb 24-9:18 AM

Evaluate

1) $\lim_{x \to 1} \dfrac{\frac{1}{x+3} - \frac{1}{4}}{x-1} = \dfrac{\frac{1}{1+3} - \frac{1}{4}}{1-1} = \dfrac{\frac{1}{4}-\frac{1}{4}}{0} = \dfrac{0}{0}$ I.F.

LCD: $(x+3)\cdot 4$

$$= \lim_{x \to 1} \frac{4\cancel{(x+3)}\cdot\frac{1}{\cancel{x+3}} - 4(x+3)\cdot\frac{1}{\cancel{4}}}{4(x+3)(x-1)} = \lim_{x \to 1} \frac{\overbrace{4-x-3}^{\overset{-1}{1-x}}}{4(x+3)(x-1)}$$

$$= \lim_{x \to 1} \frac{-1}{4(x+3)} = \boxed{\frac{-1}{16}}$$

2) $\lim_{x \to 25} \dfrac{\sqrt{x}-5}{x-25}$ Hint: Rationalize the numerator

$$= \frac{\sqrt{25}-5}{25-25} = \frac{5-5}{0} = \frac{0}{0} \text{ I.F.}$$

$(A-B)(A+B) = A^2 - B^2$

$$= \lim_{x \to 25} \frac{(\overset{A}{\sqrt{x}} \overset{-B}{-5})(\overset{A}{\sqrt{x}} \overset{+B}{+5})}{(x-25)(\sqrt{x}+5)} = \lim_{x \to 25} \frac{(\sqrt{x})^2 - (5)^2}{(x-25)(\sqrt{x}+5)}$$

$$= \lim_{x \to 25} \frac{\cancel{x-25}}{\cancel{(x-25)}(\sqrt{x}+5)}$$

$$= \lim_{x \to 25} \frac{1}{\sqrt{x}+5} = \frac{1}{\sqrt{25}+5}$$

$$= \frac{1}{5+5}$$

$$= \boxed{\frac{1}{10}}$$

Feb 24-9:30 AM

Given $f(x) = x^2 - 4x + 5$

Evaluate $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 4(x+h) + 5 - x^2 + 4x - 5}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-4x} - 4h \cancel{+5} \cancel{-x^2} \cancel{+4x} \cancel{-5}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h-4)}{\cancel{h}} = \lim_{h \to 0} (2x+h-4)$$

$$= 2x + 0 - 4 = \boxed{2x-4}$$

Feb 24-9:47 AM

$f(x)$ is Continuous at $x=a$ when

1) $f(a)$ exists,

2) $\lim_{x \to a} f(x)$ exists, and

3) $\lim_{x \to a} f(x) = f(a)$

Is $f(x) = x^2 - 4x + 10$ cont. at $x=3$?

1) $f(3) = 3^2 - 4(3) + 10 = 9 - 12 + 10 = \boxed{7}$

2) $\lim_{x \to 3} f(x) = \lim_{x \to 3} [x^2 - 4x + 10] = 3^2 - 4(3) + 10 = \boxed{7}$

Since $\lim_{x \to 3} f(x) = f(3)$, then $f(x)$ is Cont. at $x=3$.

Feb 24-10:07 AM

Is $f(x) = \frac{1}{x}$ Continuous at $x = \frac{1}{4}$?

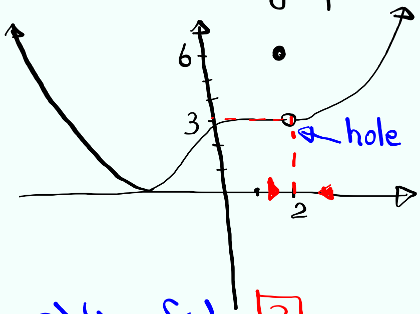
1) $f\left(\frac{1}{4}\right) = \frac{1}{\frac{1}{4}} = 4$

2) $\lim_{x \to \frac{1}{4}} f(x) = \lim_{x \to \frac{1}{4}} \frac{1}{x} = \frac{1}{\frac{1}{4}} = 4$

Since $\lim_{x \to \frac{1}{4}} f(x) = f\left(\frac{1}{4}\right)$, then $f(x)$ is Cont. at $x = \frac{1}{4}$.

Feb 24-10:11 AM

Consider the graph of $f(x)$ below



1) $\lim\limits_{x \to 2^+} f(x) = 3$

2) $\lim\limits_{x \to 2^-} f(x) = 3$

3) $\lim\limits_{x \to 2} f(x) = \boxed{3}$

4) $f(2) = 6$

5) Discuss continuity of $f(x)$ at $x=2$.

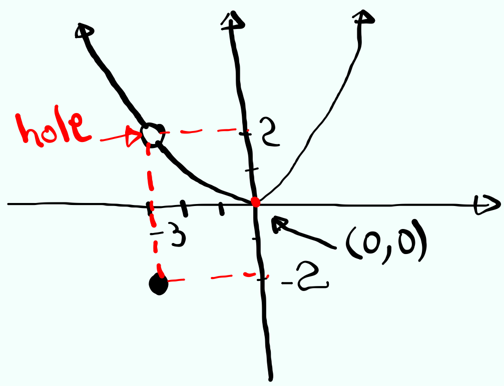
Not continuous at $x=2$, why?

$$\lim_{x \to 2} f(x) \neq f(2)$$

$$3 \neq 6$$

Feb 24-10:15 AM

Consider the graph of $f(x)$ below



1) All intercepts ← ordered-Pairs

x-Int
Y-Int
$(0,0)$

2) $\lim\limits_{x \to -3} f(x) = 2$

3) $f(-3) = -2$

4) Discuss continuity at $x=-3$.

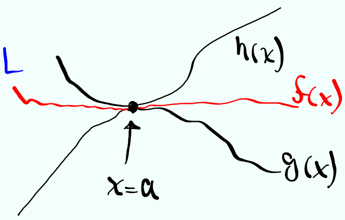
NO, $\lim\limits_{x \to -3} f(x) \neq f(-3)$

Feb 24-10:21 AM

$\boxed{\text{Squeeze Thrm}}$

If $g(x) \le f(x) \le h(x)$ over an interval that contains $x = a$ and $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$

then $\lim_{x \to a} f(x) = L$



Suppose $\cos x \le f(x) \le x^2 + 1$, find $\lim_{x \to 0} f(x)$

$\lim_{x \to 0} \cos x = \cos 0 = 1$

$\lim_{x \to 0} (x^2+1) = 0^2 + 1 = 1$

By Squeeze Thrm

$\lim_{x \to 0} f(x) = 1$

Feb 24-10:27 AM

Suppose $\sin(x-4) \le f(x) \le \sqrt{x} - 2$

Find $\lim_{x \to 4} f(x)$.

$\lim_{x \to 4} \sin(x-4) = \sin(4-4) = \sin 0 = 0$

$\lim_{x \to 4} (\sqrt{x} - 2) = \sqrt{4} - 2 = 2 - 2 = 0$

By S.T.

$\lim_{x \to 4} f(x) = 0$

Feb 24-10:33 AM

$$\boxed{\lim_{h\to 0}\frac{\sin h}{h}=1} \quad \& \quad \boxed{\lim_{h\to 0}\frac{1-\cos h}{h}=0}$$

Evaluate $\displaystyle\lim_{x\to 0}\frac{3\sin 3x}{3x}$

$$=\lim_{x\to 0}3\cdot\lim_{x\to 0}\frac{\sin 3x}{3x}$$

$$=3\cdot 1=\boxed{3}$$

Feb 24-10:37 AM

Evaluate

$$\lim_{x\to 0}\frac{\sin 3x}{\sin 5x}=\frac{\sin 3(0)}{\sin 5(0)}=\frac{\sin 0}{\sin 0}=\frac{0}{0}\ \text{I.F.}$$

$$=\lim_{x\to 0}\frac{\frac{3\sin 3x}{3x}}{\frac{5\sin 5x}{5x}}=\frac{\lim_{x\to 0}\frac{3\sin 3x}{3x}}{\lim_{x\to 0}\frac{5\sin 5x}{5x}}=\frac{3\lim_{x\to 0}\frac{\sin 3x}{3x}}{5\lim_{x\to 0}\frac{\sin 5x}{5x}}$$

$$=\frac{3\cdot 1}{5\cdot 1}=\boxed{\frac{3}{5}}$$

Feb 24-10:40 AM

Evaluate

$$\lim_{x\to 2}\frac{\sin(x-2)}{x^2-4} = \frac{\sin(2-2)}{2^2-4} = \frac{\sin 0}{0} = \frac{0}{0}$$

I.F.

$$= \lim_{x\to 2}\frac{\sin(x-2)}{(x+2)(x-2)} = \lim_{x\to 2}\left[\frac{1}{x+2}\cdot\frac{\sin(x-2)}{x-2}\right]$$

$$= \boxed{\lim_{x\to 2}\frac{1}{x+2}} \cdot \cancel{\lim_{x\to 2}\frac{\sin(x-2)}{x-2}}\ 1$$

$$= \frac{1}{2+2}\cdot 1$$

$$= \boxed{\frac{1}{4}}$$

Feb 24-10:45 AM

Evaluate

$$\lim_{x\to 2}\frac{1-\cos(x-2)}{x-2} = \frac{1-\cos(2-2)}{2-2} \rightarrow = \boxed{0}$$

$$= \frac{1-\cos 0}{0} = \frac{1-1}{0} = \frac{0}{0}$$

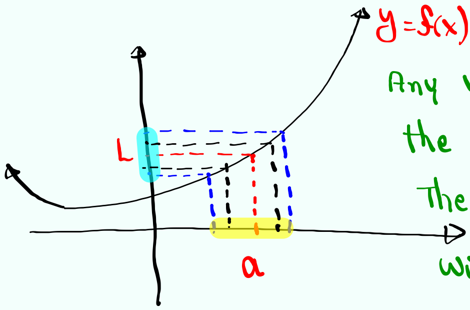
I.F.

Recall

$$\lim_{h\to 0}\frac{\sin h}{h} = 1 \quad \text{\&} \quad \lim_{h\to 0}\frac{1-\cos h}{h} = 0$$

Feb 24-10:52 AM

Precise Definition of limit:



Any value chosen from the yellow area, the function value will be in the blue area.

For any $\varepsilon > 0$, there is a $\delta > 0$ such that

Epsilon Delta

$$|f(x) - L| < \varepsilon \quad \text{whenever} \quad |x - a| < \delta$$

$$\lim_{x \to a} f(x) = L$$

Feb 24-10:56 AM

Prove $\lim_{x \to 1} (2x + 5) = 7$.

$f(x) = 2x + 5$ 1) Let's verify the limit.

$a = 1$ $\lim_{x \to 1} (2x+5) = 2(1) + 5 = 2 + 5 = 7$ ✓

$L = 7$

For $\varepsilon > 0$, there is a $\delta > 0$ such that

$$|f(x) - L| < \varepsilon \quad \text{whenever} \quad |x - a| < \delta$$

$$|2x + 5 - 7| < \varepsilon \quad \text{whenever} \quad |x - 1| < \delta$$

$$|2x - 2| < \varepsilon \quad '' \quad |x - 1| < \delta$$

$$|2(x - 1)| < \varepsilon \quad '' \quad |x - 1| < \delta$$

$$2|x - 1| < \varepsilon \quad '' \quad |x - 1| < \delta$$

$$|x - 1| < \frac{\varepsilon}{2} \quad '' \quad |x - 1| < \delta$$

$$\boxed{\delta = \frac{\varepsilon}{2}}$$

$\varepsilon = 1 \to \delta = \frac{1}{2}$

$\varepsilon = 2 \to \delta = 1$

$\varepsilon = \frac{1}{2} \to \delta = \frac{1}{4}$

Feb 24-11:02 AM

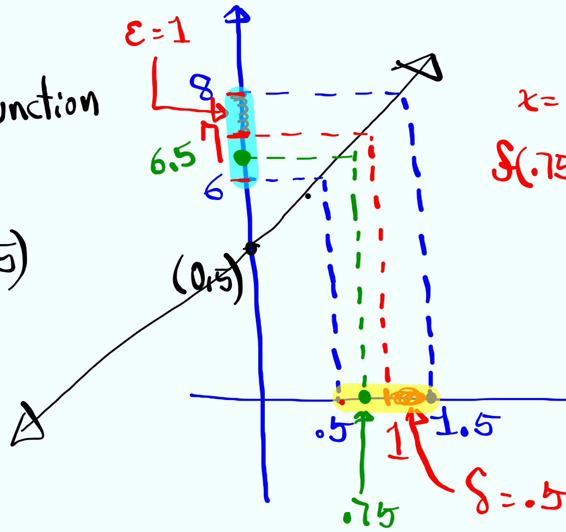
$f(x) = 2x+5$

Linear function

$m = 2$

Y-Int$(0,5)$

Not Scaled



$\varepsilon = 1 \rightarrow \delta = \frac{1}{2}$

$x = .75$

$f(.75) = 2(.75)+5$

$= 1.5 + 5$

$= 6.5$

$x = 1.4$

$f(1.4) = 2(1.4)+5$

$= 2.8 + 5$

$= 7.8$

Feb 24-11:09 AM